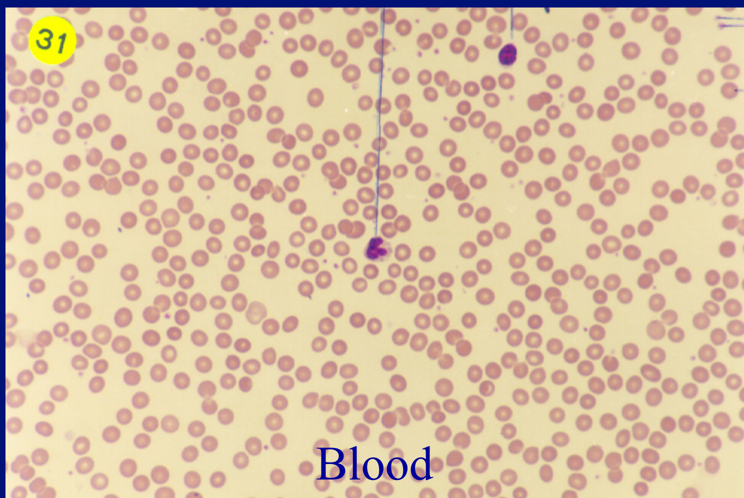
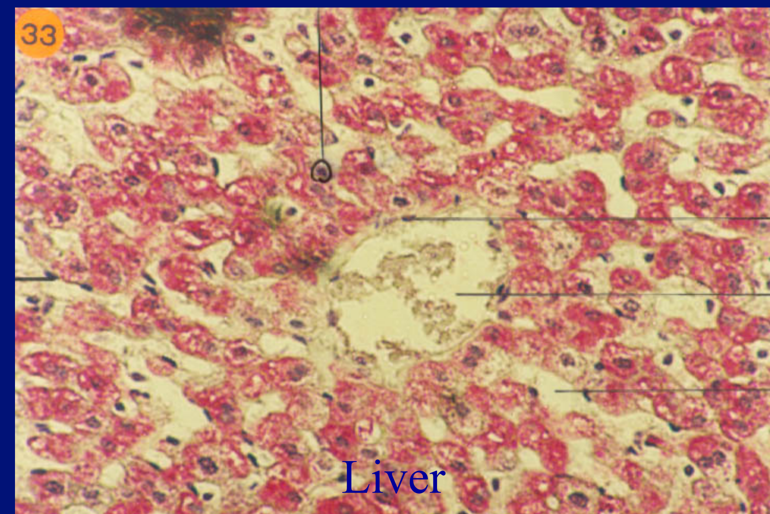
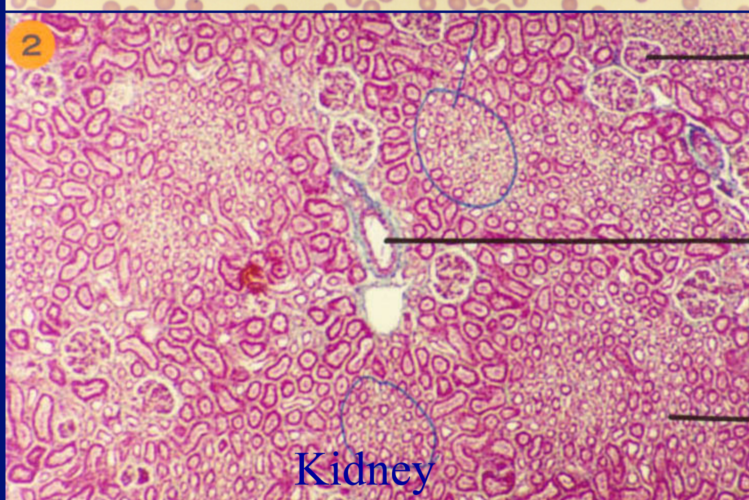
Chapter 3:
Scattering, Attenuation and Speckle

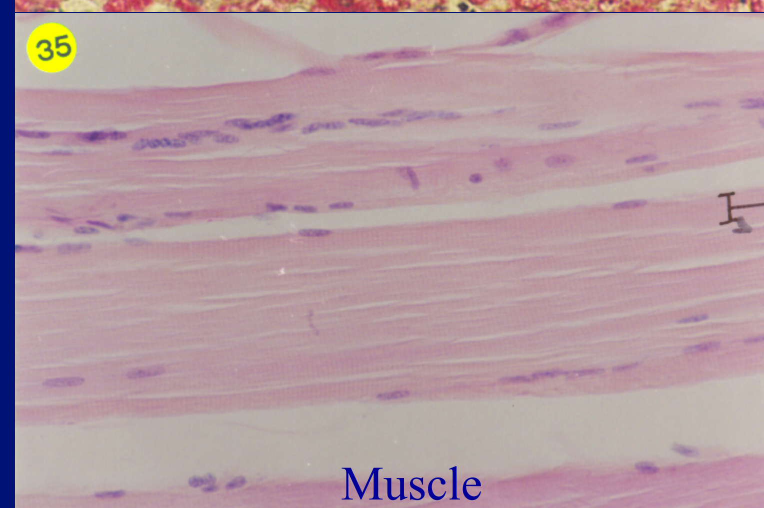
Scattering

Histology

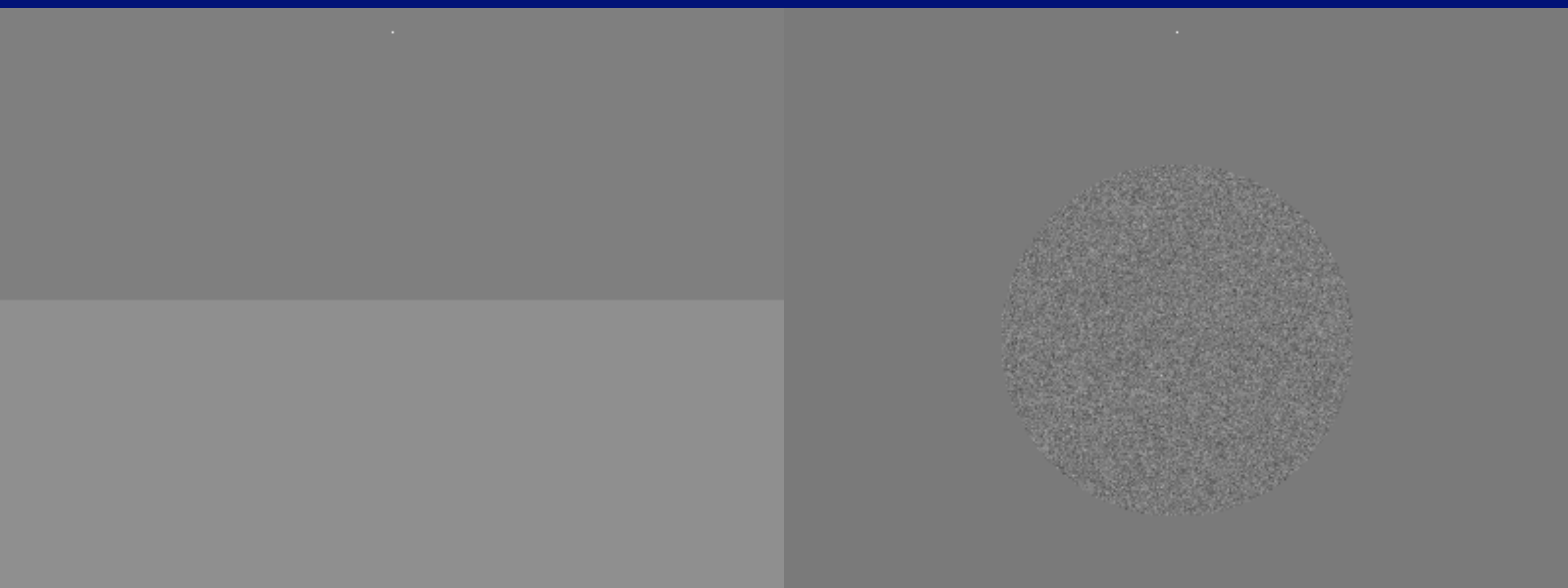








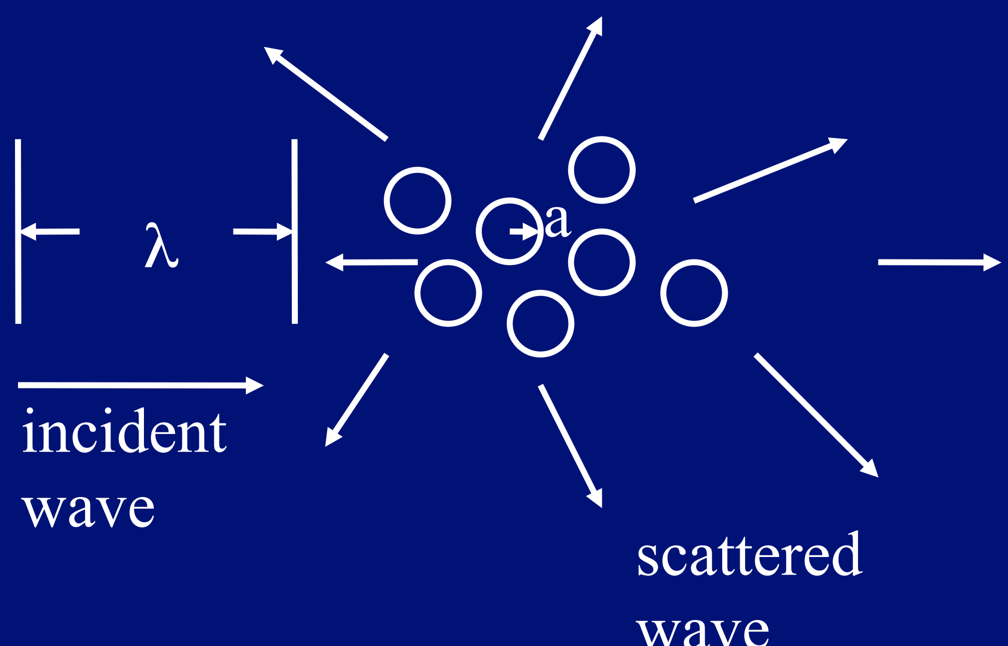
Reflection vs. Scattering



Scattering

- (Specular) Relfection vs. (Rayleigh) Scattering.
- Angular scattering vs. Back-scattering.

- Optical : ka>>1
- Rayleigh : ka<<1.
- Oscillatory : in between.



Scatter Parameters

- Scatter cross section (σ_s):
  - Total scattered power/Incident energy.
- Backscatter cross section (σ_b).
- Backscatter coefficient (ε):
  - σ_b per unit volume of scatterers.
  - ε normalized to solid angle (sr^{-1}).

Scattering Properties

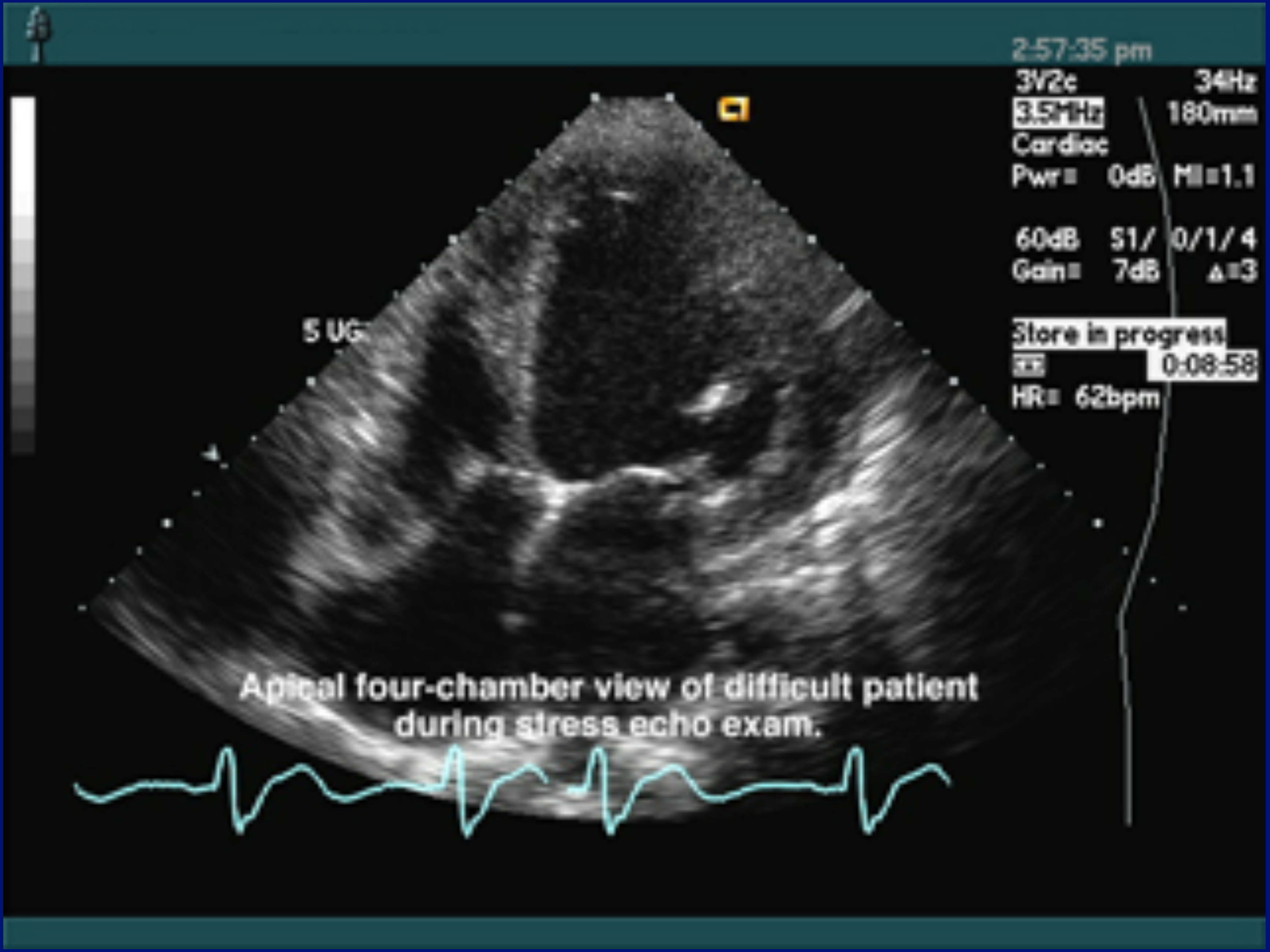
- Rayleigh scattering (ignoring secondary scattering):

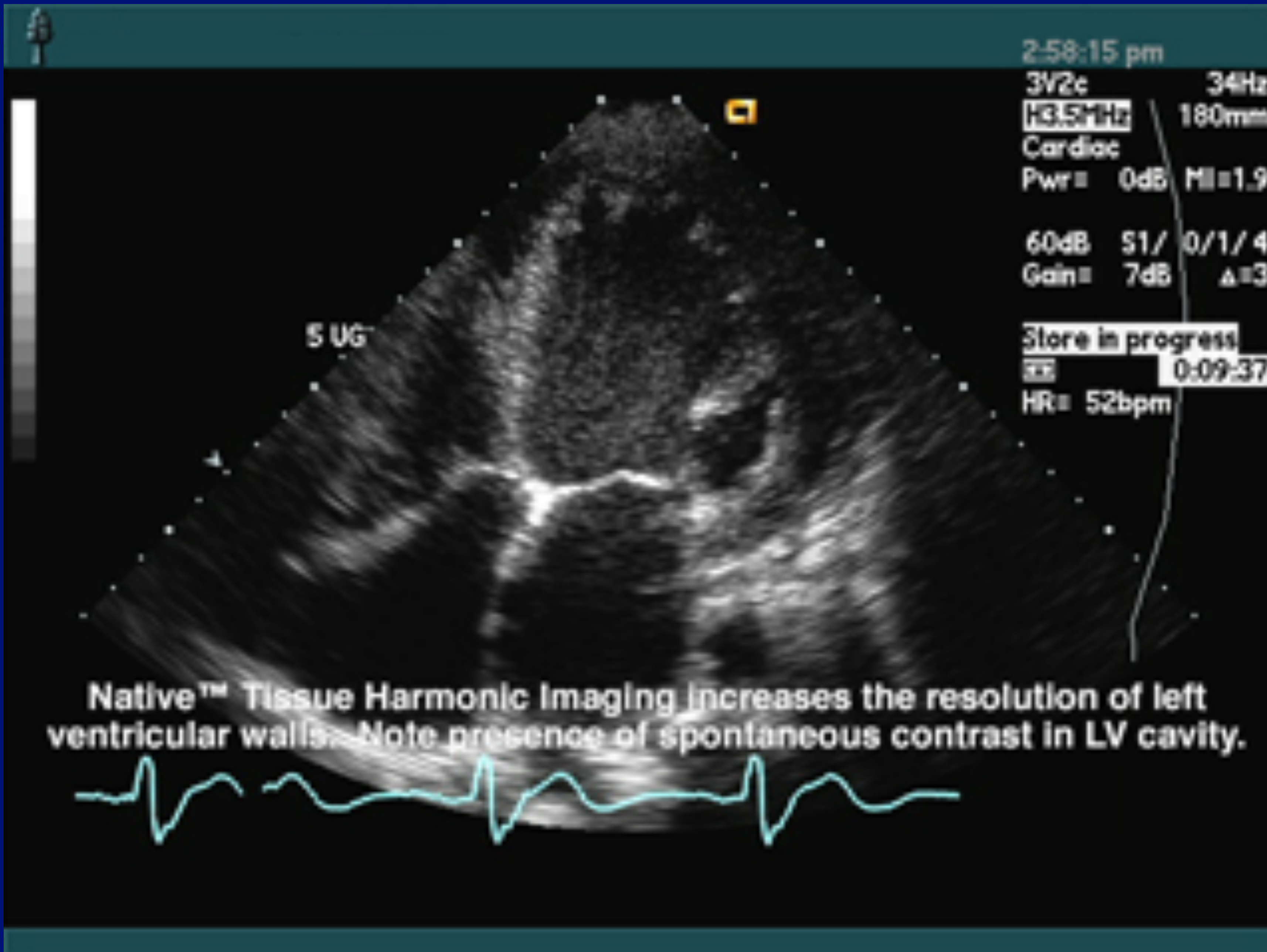
$$\sigma_s \propto k^4 a^6$$

- Determing factors:
  - Size and structure.
  - Cell, blood vessel and ductal network.
- Roughly Speaking:
  - Blood: f^4.
  - Myocardium: f^3.
  - Other soft tissue: $f^{1.5\text{-}2.5}$.

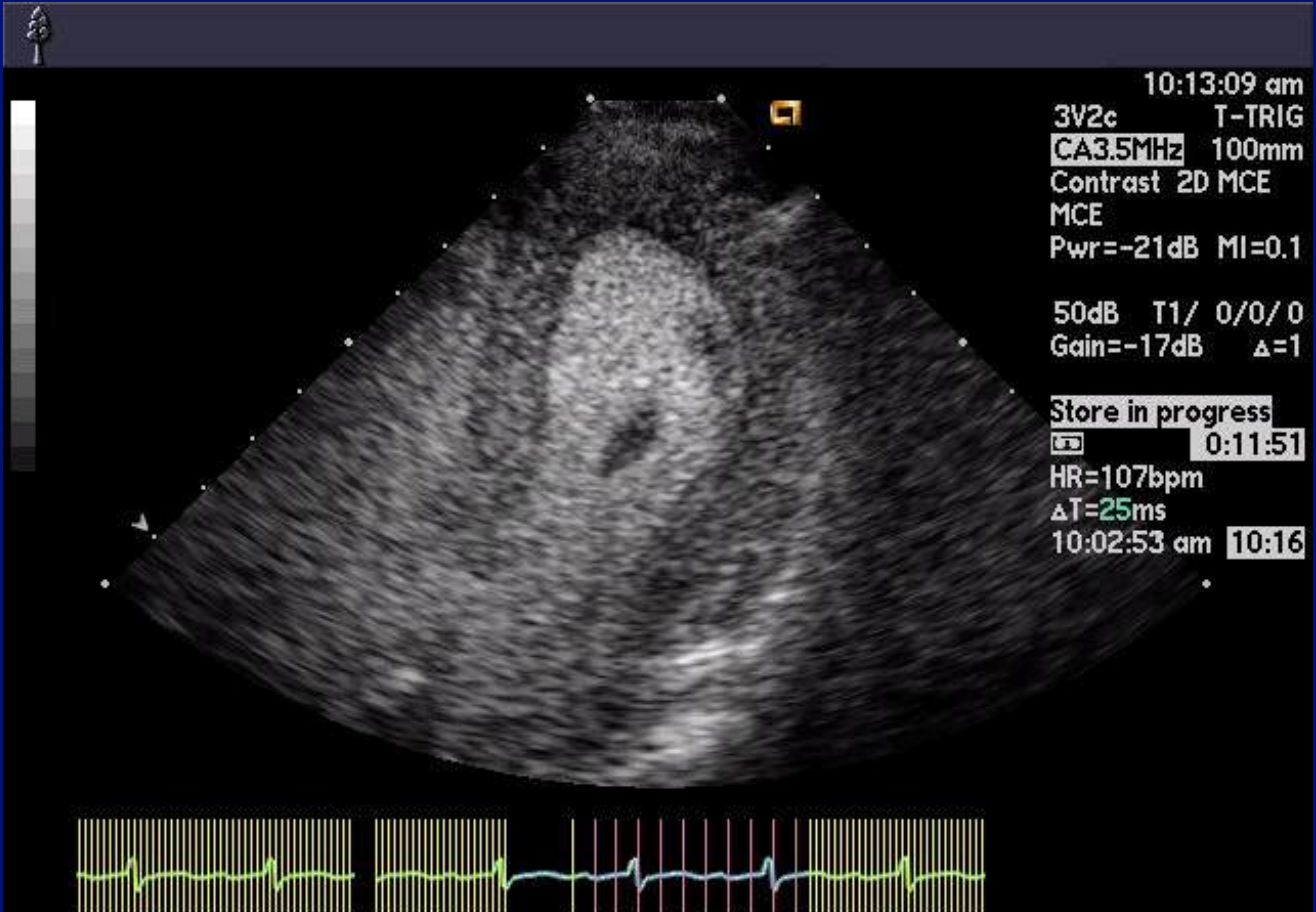
Scattering Properties

Frequency (MHz)	$\varepsilon(mm^{-1})$ heart tissue	$\varepsilon(mm^{-1})$ blood
2.5	$4.3 \cdot 10^{-5}$	$0.5 \cdot 10^{-6}$
3.75	$1.5 \cdot 10^{-4}$	$2.6 \cdot 10^{-6}$
5.0	$5.0 \cdot 10^{-4}$	$8.2 \cdot 10^{-6}$


2:57:35 pm
3V2c
34Hz
3.5MHz
180mm
Cardiac
Pwr= 0dB
MI=1.1
60dB
S1/ 0/1/ 4
Gain= 7dB
Δ=3
Store in progress
0:08:58
HR= 62bpm
5 UG
Apical four-chamber view of difficult patient during stress echo exam.



Native™ Tissue Harmonic Imaging increases the resolution of left ventricular walls. Note presence of spontaneous contrast in LV cavity.



Real Time CCI Myocardial Conrast Enhancement

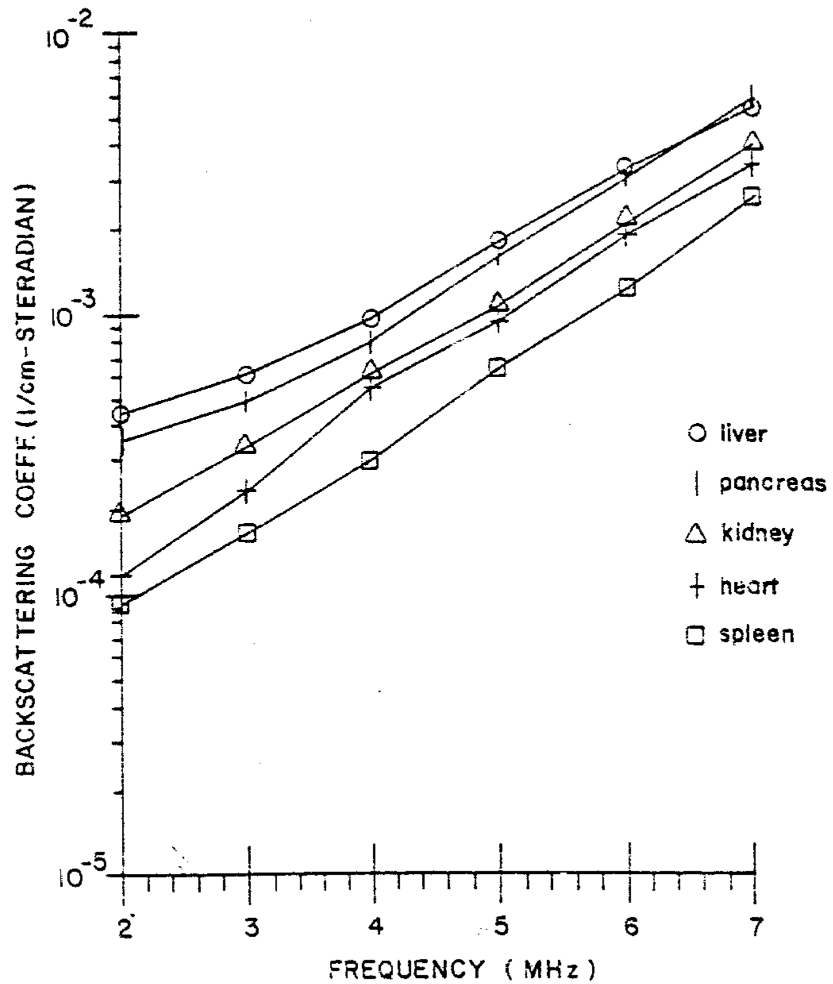


Figure 75 Backscattering coefficient of bovine tissues as a function of frequency.

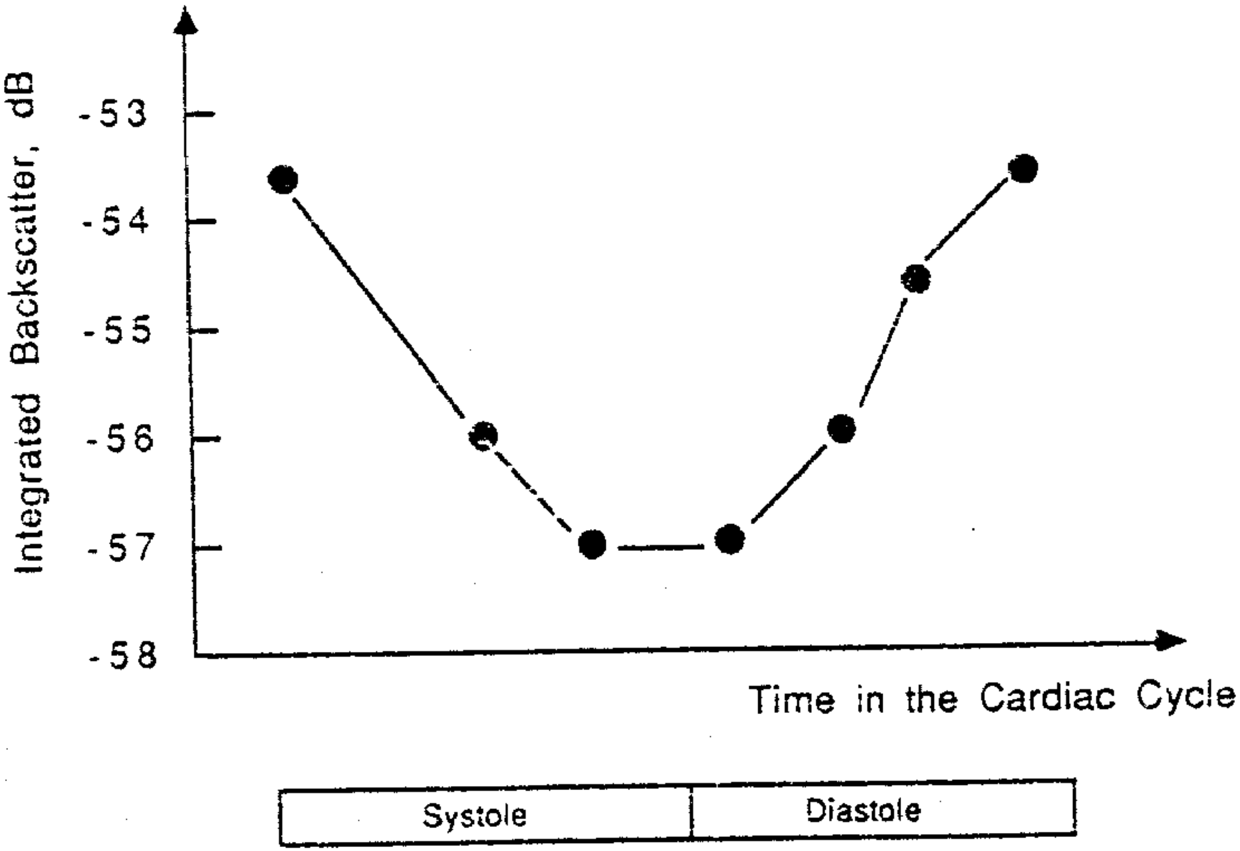


Figure 77 Integrated backscatter defined as the averaged backscatter coefficient over a frequency band relative to that from a flat reflector of canine myocardium measured *in vivo* as a function of cardiac cycle. (From Miller *et al.*, 1985).

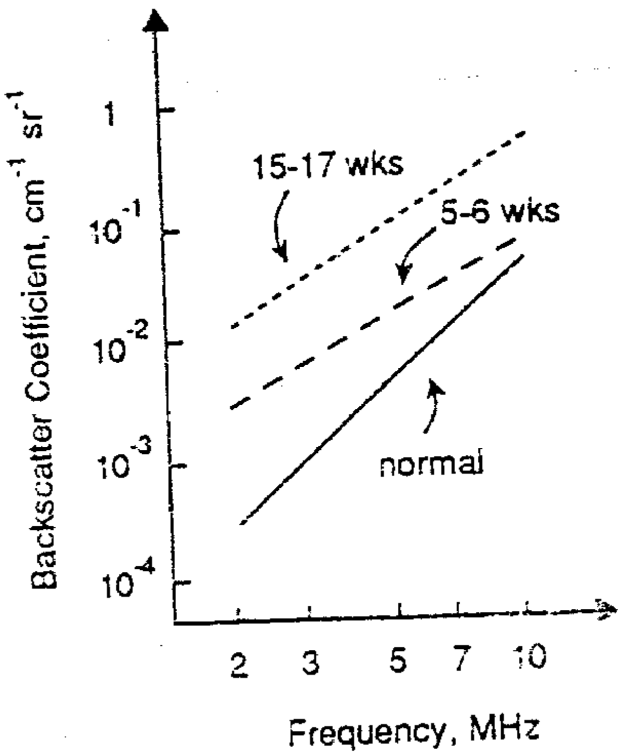


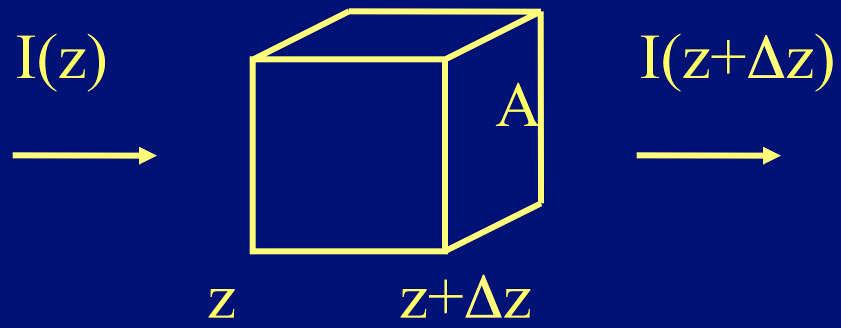
Figure 76 Backscattering coefficient of canine myocardium as a function of frequency. Solid line, normal; dashed line, 5–6 weeks after coronary occlusion; dotted line, 15–17 weeks after coronary occlusion. (From O'Donnell *et al.*, 1981).

Attenuation

Attenuation

- Sources of energy loss:
  - Reflection and scattering.
  - Relaxation.
- Relaxation:
  - Pressure change and volume change are not in phase.
  - Product of absorption and wavelength are roughly constant.
- Fundamental limitations of penetration:
  - Attenuation.
  - Safety requirements.

Attenuation



$$A \times I(z+\Delta z) = A \times I(z) - 2\beta A \times I(z)\Delta z$$

$$-\frac{\partial I(z)}{\partial z} = 2 \times \beta I(z)$$

$$I(z) = I_0 e^{-2\beta z}$$

$$\beta = \alpha f$$

Attenuation

$$H(z,f) = e^{-(\alpha f z + j2\pi f z / c)}$$

$$I(z,f) = I_0 \left| H(z,f) \right|^2 = I_0 e^{-2\alpha f z}$$

$$-10\log_{10}\left(\frac{I(z,f)}{I_0}\right) = 20(\log_{10} e)\alpha f z = 8.69\alpha f z$$

$$\alpha_{dB} = 8.69\alpha_{nepers}\,.$$

Table V

Attenuation coefficients of biological tissues and pertinent materials

Material	Attenuation coefficient (np/cm at 1 MHz at 20°C)
Air	1.38
Aluminum	0.0021
Plexiglas	0.23
Water	0.00025
Fat	0.06
Blood	0.02
Myocardium (perpendicular to fiber)	0.35
Liver	0.11
Kidney	0.09
Skull bone	1.30

Attenuation

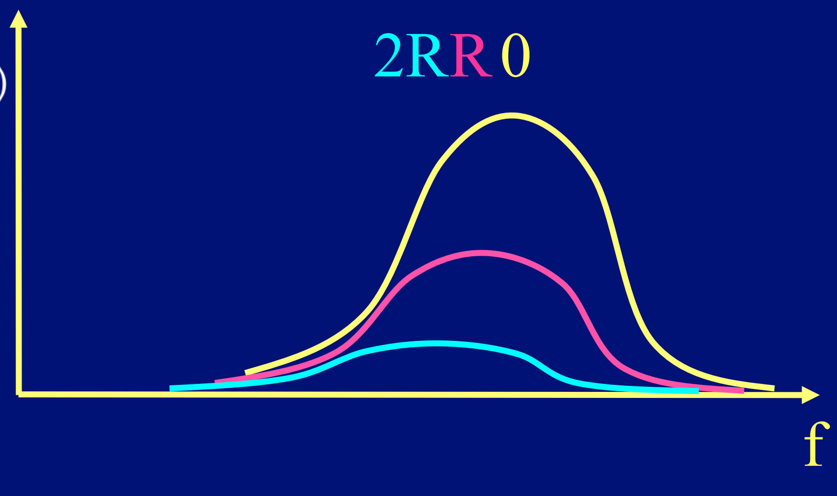
- Assuming a Gaussian signal:

$$\left|S_t(f)\right|^2 = e^{-\left(\frac{f-f_0}{\sigma}\right)^2}$$

$$\left|S_r(R,f)\right|^2 = \left|S_t(f)\right|^2 e^{-4\alpha Rf} = e^{-\left(\frac{f-f_0}{\sigma}\right)^2 - 4\alpha Rf}$$

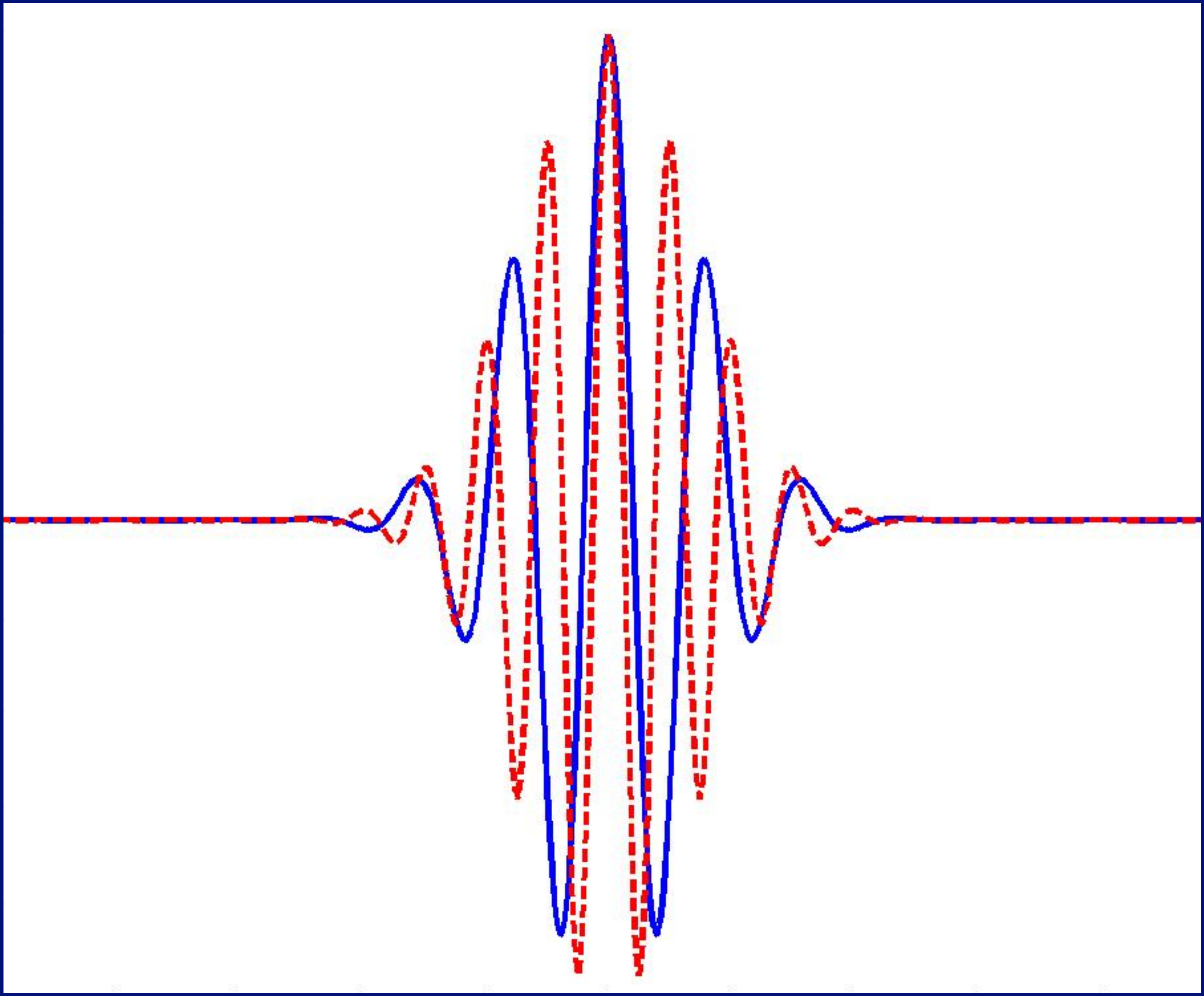
$$\left|S_r(R,f)\right|^2 = e^{-\left(\frac{f-f_1}{\sigma}\right)^2} e^{-4\alpha R(f_0-\sigma^2\alpha R)}$$

$$f_1 = f_0 - 2\sigma^2\alpha R.$$



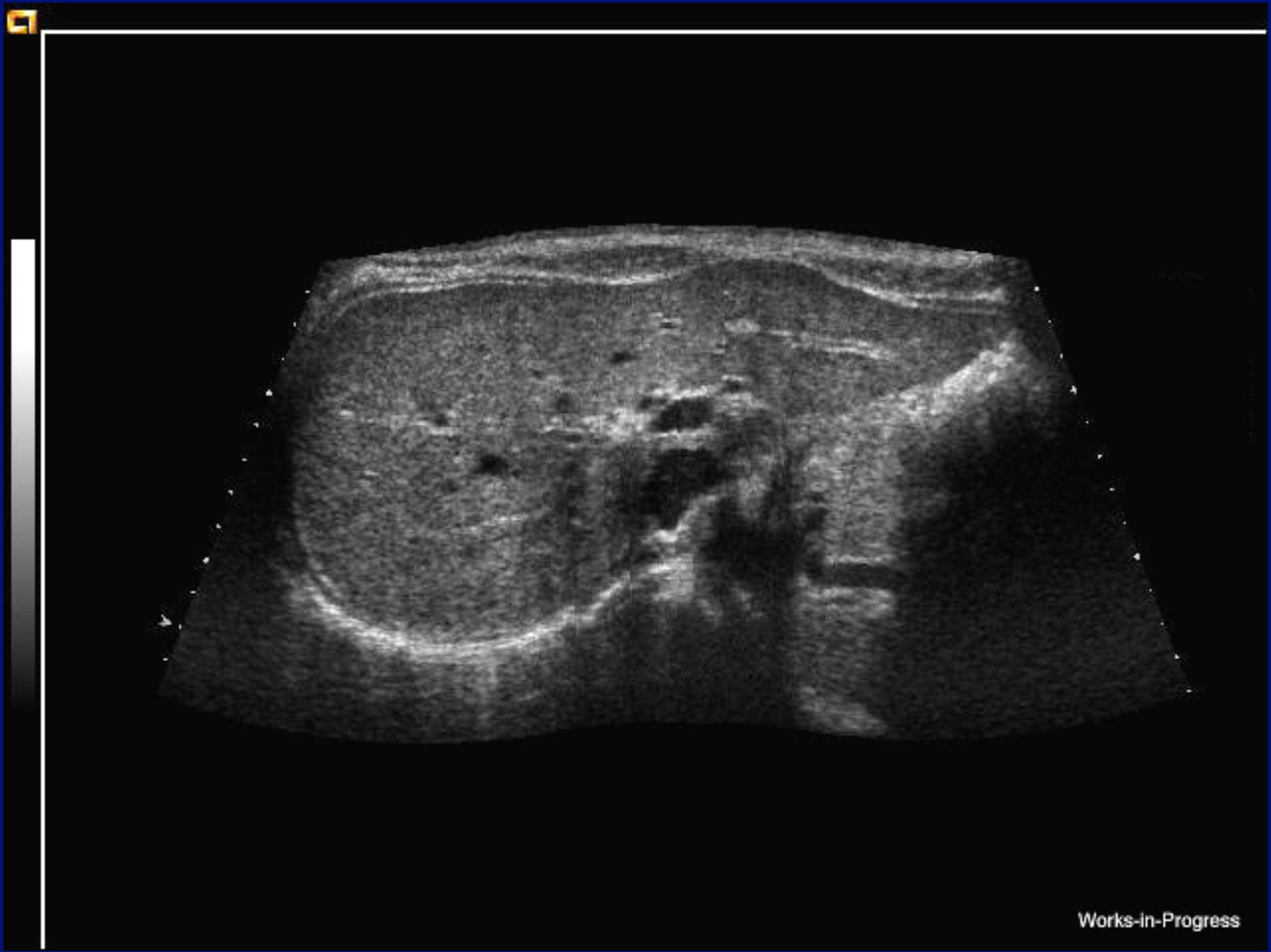
Attenuation on Pulse Shape

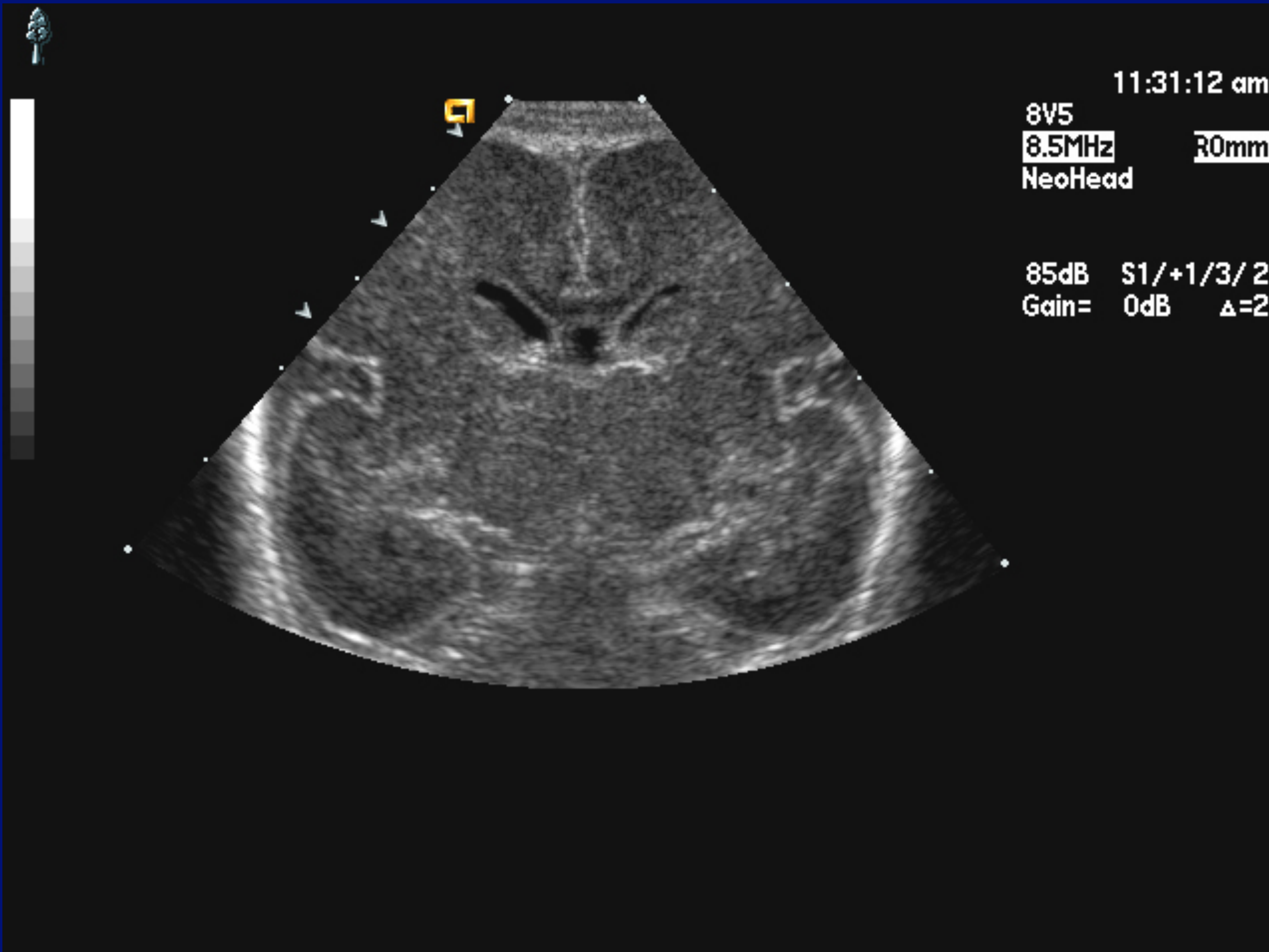
- Center frequency downshift → Lateral resolution decreases with depth.
- The downshift is proportional to:
  - $Bandwidth^2$.
  - Attenuation coefficient.
- Absolute bandwidth is un-changed → Axial resolution is un-affected.
- Tradeoff between lateral and axial resolution.



Speckle

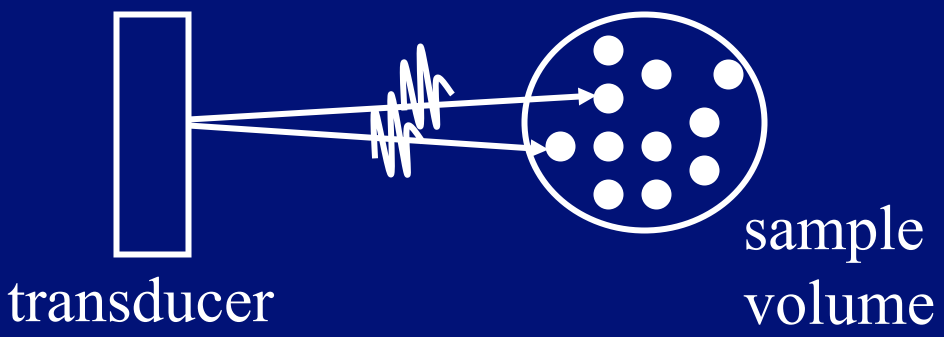

2:07:06 pm
8C4
H8.0MHz
75dB S1/+1/1/4
Gain= -6dB ▵=2


Works-in-Progress


11:31:12 am
8V5
8.5MHz
R0mm
NeoHead
85dB S1/+1/3/ 2
Gain= 0dB Δ=2

Speckle Formation

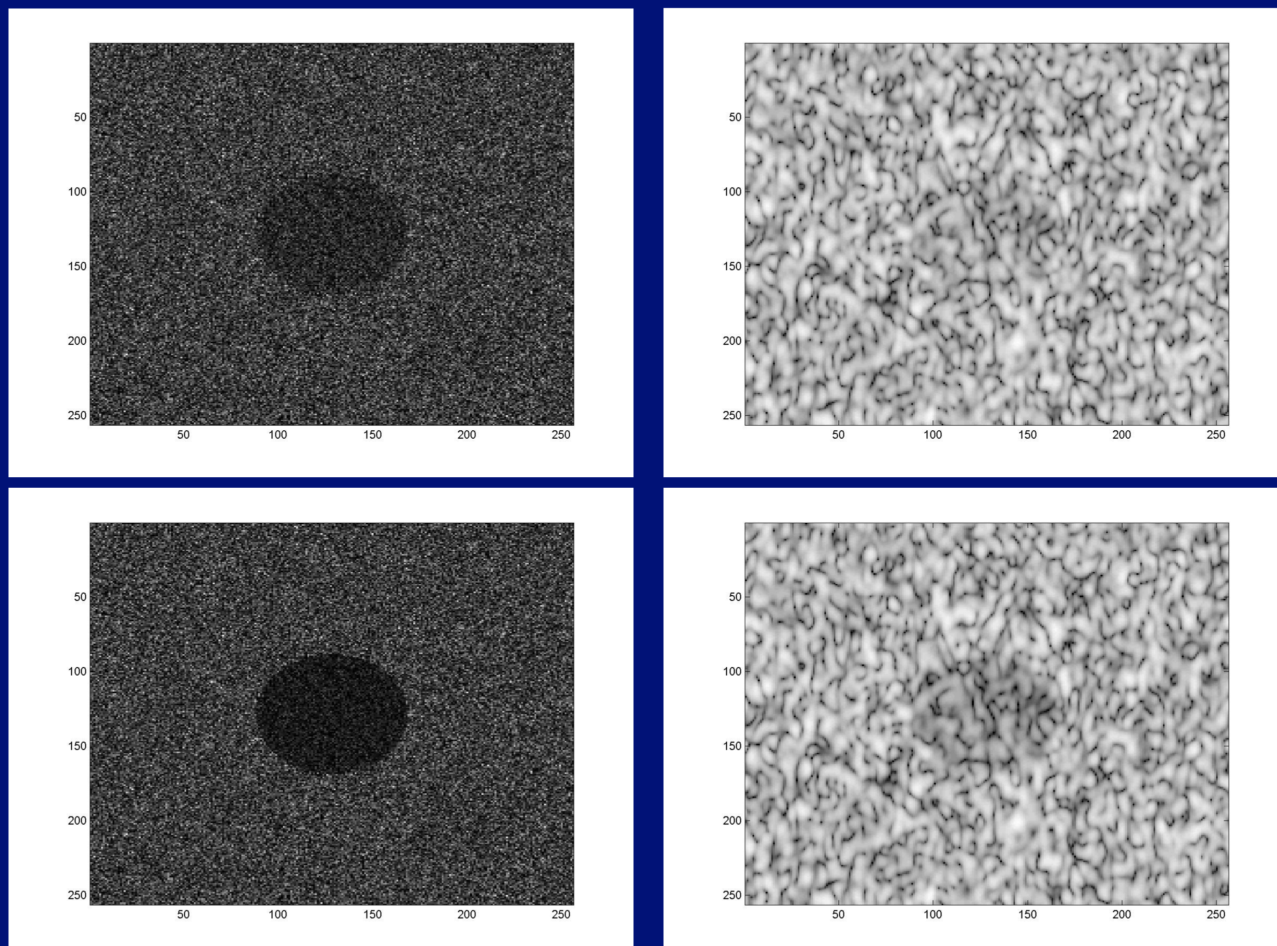
- Speckle results from coherent interference of un-resolvable objects.



Speckle Formation

- In diagnostic ultrasound, the size of tissue micro-structures is often much smaller than a typical wavelength.
- Pulse-echo ultrasonic images are formed using the phase information.
- Speckle appears as brightness variations and obscure the underlying information.

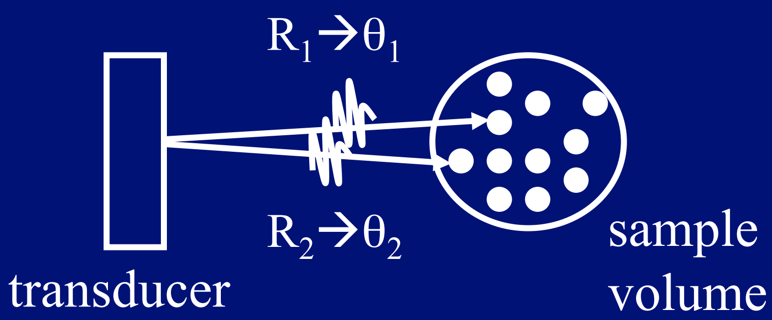
Speckle Noise

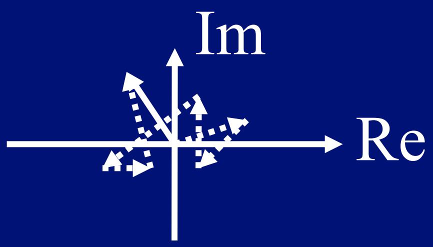


Speckle Noise

- Coherent sum of random signals from sound scatterers in a resolution cell.
- Brightness variations are independent of tissue properties.
- Multiplicative noise.
- Fundamental limitation of contrast resolution.

Speckle First-Order Statistics





$$\mathrm{Re}\{A\} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |a_k| \cos\theta_k$$

$$\mathrm{Im}\{A\} = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |a_k| \sin\theta_k$$

$$p_{\mathrm{Re}\{A\}.\mathrm{Im}\{A\}} = \frac{1}{2\pi\sigma^2} e^{-\frac{\mathrm{Re}\{A\}^2 + \mathrm{Im}\{A\}^2}{2\sigma^2}}$$

$$\sigma^2 = \frac{1}{N} \sum_{k=1}^{N} \frac{|a_k|^2}{2}$$

Speckle First-Order Statistics

$$p_I = \frac{1}{2\sigma^2} e^{-\frac{I}{2\sigma^2}}$$

$$p_E = \frac{E}{\sigma^2} e^{-\frac{E^2}{2\sigma^2}}$$

$$SNR_I \equiv \frac{\langle I \rangle}{\sigma_I} = 1$$

$$SNR_E \equiv \frac{\langle I \rangle}{\sigma_E} = \frac{\left(\pi\sigma^2/2\right)^{1/2}}{\left(\left(4-\pi\right)\sigma^2/2\right)^{1/2}} \approx 1.91$$

Speckle First-Order Statistics

- On a log display:

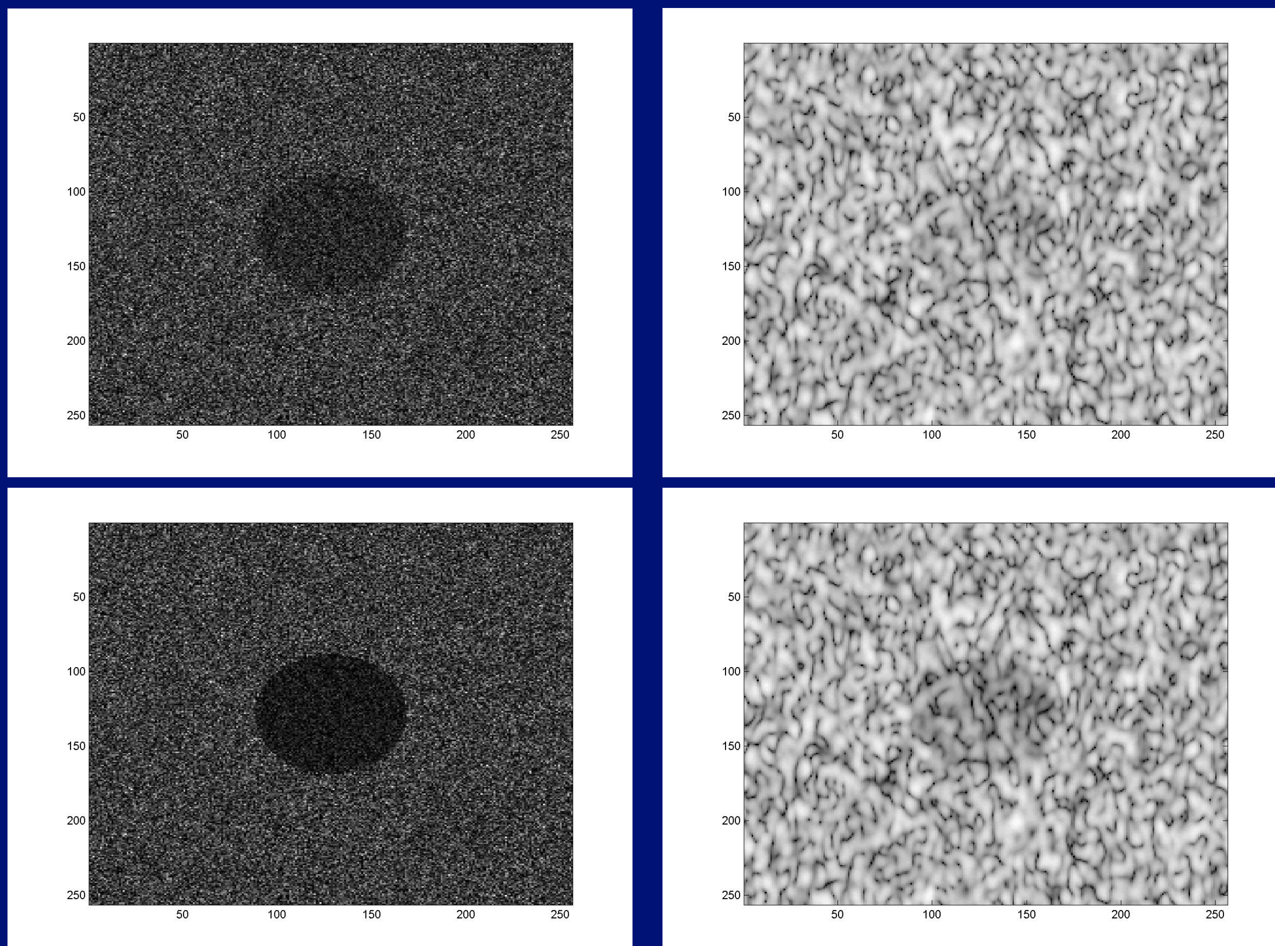
$$D(dB) = f(I) \equiv 10\log_{10}\left(\frac{I}{I_0}\right)$$

$$D = f(\langle I \rangle) + \left(I - \langle I \rangle\right) f'\left(\langle I \rangle\right) + R$$

$$\sigma_D^{\,2} \approx f'(\langle I \rangle)^2 \sigma_I^{\,2} = \left(\frac{10}{\ln 10}\right)^2 \frac{\sigma_I^{\,2}}{\langle I \rangle^2}$$

$$\sigma_D \approx 4.34(dB)$$ ←Fundamental Limitation of Contrast Resolution

Speckle Noise



-3 dB

-6 dB

Homework #1

- Computer Homework #1: Speckle Statistics
- Due 5:00pm 3/20/2012 by emailing to paichi@ntu.edu.tw
- Please use a single MATLAB program for the entire homework.

Problem 1

1. Create an array of 10,000 complex data with the following statistics:

- Uniform distribution of amplitude in [0, 1].
- Uniform distribution of phase in [0, 2π].
- Plot the histograms of the amplitude and intensity of the above data.

Problem 2

2. Create a new array of N data points based on the original array (N=10,000, 5,000, 2,000, 1,000 and 500). The ith point of the new array is the sum of M consecutive data points (M=1, 2, 5, 10 and 20) of the original array (from (i-1)*M+1 to i*M). Calculate and plot the ratio of the mean to the standard deviation of the amplitude and intensity arrays as a function of M.

Problem 3

3. Repeat 1 and 2 by making the amplitude distribution normal with (0, 1).

Problem 4

4. Repeat 1 and 2 by making the phase distribution normal with (0, 1).

Problem 5

5. (bonus, not required) Use the program to investigate any issues relevant to this topic (speckle statistics).